A compact representation of magnetotelluric responses for two-layer models

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Abstract

In this paper the locations where $\rho_{\text{app}} = \rho_1$ and $\varphi = \pi/4$ and where these parameters reach an extreme value in two-layer magnetotelluric (MT) sounding curves are summarized in an extremely compact form. The key parameters over two-layer models with conductivities $\sigma_1$, $\sigma_2$ and upper layer thickness $h$ are the real $S$ and $\alpha$, where $S$ is the conductivity contrast and $\alpha$ is the distance between the observation site and the conductivity interface, normalized to the half skindepth in the first layer. If the impedance components, various resistivity definitions ($\rho_{\text{Re}Z}$, $\rho_{\text{Im}Z}$ and $\rho_{|Z|}$, based on different parts of the complex impedance $Z$) and the magnetotelluric phase $\varphi$ are derived as a function of $S$ and $\alpha$, then the conditions for the apparent resistivity $\rho_{\text{app}}$ and the phase $\varphi$ are that they either satisfy $\rho_{\text{app}} = \rho_1$, and $\varphi = \pi/4$ or attain extreme values which can be given in terms of simple algebraic equations between $S$ and $\alpha$. All equations are valid for observation sites at any depth $0 \leq z \leq h$ in the first layer. The set of equations, presented in a tabular form, may make it possible to determine a layer boundary from the short period part of the sounding curves, in particular the $\rho_{\text{Re}Z}$ and the $\varphi_{\text{MT}}$ curves.

Introduction

In the history of magnetotellurics, a number of alternative apparent resistivity definitions have been proposed. In this paper only apparent resistivity definitions in the frequency domain are considered. The traditional definition (denoted here as $\rho_{|Z|}$ given by Cagniard (1953)) is based on the absolute value of the magnetotelluric impedance $Z$. Schmucker (1970) proposed using the apparent resistivity based on the real part of the impedance. Properties of different apparent resistivity definitions ($\rho_{|Z|}$, $\rho_{\text{Re}Z}$, $\rho_{\text{Im}Z}$, $\rho_{\text{Re}Z^2}$, $\rho_{\text{Im}Z^2}$ and $\rho_{|Z|}$) have been compared numerically by Spies and Eggers (1986). In their study they used two criteria: (1) minimum oscillation preceding the transition, and (2) maximum rate of convergence to the underlying resistivity. They found that the apparent resistivity derived from $\text{Re}Z$ exhibits the most favourable behaviour and the apparent resistivity defined from $\text{Im}Z$ is the least useful.

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Different apparent resistivity definitions in terms of in-phase and out-of-phase current distributions at depth were studied by Szarka and Fischer (1989, 1991). The most recent definition of apparent resistivity was proposed by Başokur (1994) and discussed by Szarka (1994).

The cases $\rho_{app} = \rho_1$ and $\phi = \pi/4$ take place at periods where, due to interference between the downgoing and reflected waves, the effect of subsurface layer boundaries becomes completely invisible. The oscillations of the sounding curves are defined by Zhdanov and Keller (1994) as destructive and constructive interferences.

The idea of considering how the oscillatory behaviour of different sounding parameters depends on the subsurface features arose first in 3D analogue modelling studies, where a number of unexpected phenomena were encountered (Szarka 1991). For a correct 3D interpretation in the future, apparently elementary questions of interpretation, such as the interference sites of different sounding curves, must be answered. Since this kind of analysis, quite surprisingly, has so far not been carried out, we consider here the simplest two-layer cases in a systematic way.

**Derivation of impedance elements in terms of $S$ and $\alpha$**

The magnetotelluric impedances $Z_1$ and $Z_2$ in the first and the second layers of a one-dimensional (1D) two-layer model with conductivities $\sigma_1$, $\sigma_2$ and layer thickness $h$ are derived in the Appendix and presented in their conventional form in (A13) and (A14). In this section several very simple further algebraic steps will be carried out on the impedance expression (A13) by introducing the ‘reflection coefficient’ (conductivity contrast) $S$, the ‘wavelength’ $\lambda_1$ and skindepth $\delta_1$ in the first layer.

The complex wavenumber is defined according to (A5) as

$$k_1 = \sqrt{\frac{\omega \mu \sigma_1}{2}} (1 + i).$$

The wavelength $\lambda_1$ and the skindepth $\delta_1$ in medium 1 are defined as

$$\lambda_1 = \frac{2\pi}{\text{Re} k_1} = \frac{2\pi}{\sqrt{\frac{\omega \mu \sigma_1}{2}}}, \quad \delta_1 = \frac{1}{\text{Re} k_1} = \sqrt{\frac{2\rho_1}{\omega \mu}}.$$

The exponential term in (A13) can be written by using Euler’s relation as follows:

$$e^{2\pi k_1 (h-z)} = e^{\frac{2\pi \sigma_1}{\lambda_1}} e^{\frac{2\pi \sigma_2}{\lambda_1}} = e^{\alpha (\cos \alpha + i \sin \alpha)},$$

where $\alpha = \frac{4\pi (h-z)}{\lambda_1} = \frac{2(h-z)}{\delta_1}.$

From (3) it can be seen that $4\pi/\alpha$ is simply the ratio of the wavelength $\lambda_1$ in the first layer to the distance between the observation site in the first layer and the conductivity interface. (If $z = 0$, i.e. the observation site is on the surface, $4\pi/\alpha$ will give $\lambda_1/h$, the wavelength/layer thickness ratio.) In other words, $\alpha$ gives the depth of the conductivity
interface below the observation site, measured in half skindepths in the first layer. Thus, for simplicity \( \alpha \) will henceforth be called the 'depth/half-skindepth ratio'.

We also recall that the reflection coefficient \( S \) is given by

\[
S = \frac{k_1 - k_2}{k_1 + k_2} = \frac{\sqrt{\sigma_1} - \sqrt{\sigma_2}}{\sqrt{\sigma_1} + \sqrt{\sigma_2}},
\]

and always took values between \(-1\) and \(+1\). Then, starting from (A13), the impedance in medium 1 can be written as

\[
Z_1 = \frac{\ii \omega \mu}{k_1} \frac{e^{2\alpha} - S^2 - \ii (2G_1 \alpha \sin \alpha)}{e^{2\alpha} - 2G_1 \cos \alpha + S^2}.
\]

Eliminating the imaginary unit \( \ii \) and introducing the resistivity \( \rho_1 \) instead of \( 1/\sigma_1 \), we obtain

\[
Z_1 = \sqrt{\rho_1} \frac{\omega \mu}{2} \frac{e^{2\alpha} + 2G_1 \cos \alpha - S^2 + \ii (e^{2\alpha} - 2G_1 \cos \alpha - S^2)}{e^{2\alpha} - 2G_1 \cos \alpha + S^2}.
\]

In \( 6 \), \( Z_1 / \sqrt{\omega \mu \rho_1 / 2} \) depends exclusively on two real parameters: the conductivity contrast \( S \) and the 'depth/half-skindepth ratio', \( \alpha \).

From \( 6 \), the real and imaginary parts of \( Z_1 \), as well as its absolute value, can be derived:

\[
\text{Re} \ Z_1 = \sqrt{\rho_1} \frac{\omega \mu}{2} \frac{e^{2\alpha} + 2G_1 \cos \alpha - S^2}{e^{2\alpha} - 2G_1 \cos \alpha + S^2},
\]

\[
\text{Im} \ Z_1 = \sqrt{\rho_1} \frac{\omega \mu}{2} \frac{e^{2\alpha} - 2G_1 \cos \alpha - S^2}{e^{2\alpha} - 2G_1 \cos \alpha + S^2},
\]

\[
|Z_1| = \sqrt{\rho_1 \omega \mu} \frac{\sqrt{(e^{4\alpha} - 2G_1 e^{2\alpha} \cos 2\alpha + S^4)}}{e^{2\alpha} - 2G_1 \cos \alpha + S^2} = \sqrt{\rho_1 \omega \mu} \frac{\sqrt{e^{2\alpha} + 2G_1 \cos \alpha + S^2}}{e^{2\alpha} - 2G_1 \cos \alpha + S^2}.
\]

Without the term \( \sqrt{\rho_1 \omega \mu / 2} \) or \( \sqrt{\rho_1 \omega \mu} \) all expressions on the right-hand side of the above equations are dimensionless quantities of the so-called dimensionless impedance \( D_1 \), where

\[
\text{Re} \ D_1 = \frac{e^{2\alpha} + 2G_1 \cos \alpha - S^2}{e^{2\alpha} - 2G_1 \cos \alpha + S^2},
\]

\[
\text{Im} \ D_1 = \frac{e^{2\alpha} - 2G_1 \cos \alpha - S^2}{e^{2\alpha} - 2G_1 \cos \alpha + S^2},
\]

\[
|D_1| = \frac{\sqrt{2(e^{4\alpha} - 2G_1 e^{2\alpha} \cos 2\alpha + S^4)}}{e^{2\alpha} - 2G_1 \cos \alpha + S^2} = \sqrt{2} \frac{\sqrt{e^{2\alpha} + 2G_1 \cos \alpha + S^2}}{e^{2\alpha} - 2G_1 \cos \alpha + S^2}.
\]

Based on (7)-(12), similar expressions can be given for the so-called frequency-normalized impedance (introduced by Başokur 1994) as

\[ Y_1 = Z_1/(\omega \mu)^{1/2}. \]  

(13)

Equations for Re \( Y_1 \), Im \( Y_1 \) and \( |Y_1| \) are as follows:

\[ \text{Re } Y_1 = \sqrt{\rho_1} \frac{e^{2a} - S^2}{e^{2a} - 2Se^a \cos \alpha + S^2}, \]  

(14)

\[ \text{Im } Y_1 = \sqrt{\rho_1} \frac{-2Se^a \sin \alpha}{e^{2a} - 2Se^a \cos \alpha + S^2}, \]  

(15)

\[ |Y_1| = \sqrt{\rho_1} \frac{\sqrt{(e^{4a} - 2S^2 e^{2a} \cos 2\alpha + S^4)}}{e^{2a} - 2Se^a \cos \alpha + S^2} = \sqrt{\rho_1} \frac{\sqrt{e^{2a} + 2Se^a \cos 2\alpha + S^2}}{e^{2a} - 2Se^a \cos \alpha + S^2}. \]  

(16)

The three most important apparent resistivity definitions and the phase can then be written as

\[ \rho_{Re Z} = \frac{2}{\omega \mu} (\text{Re } Z_1)^2 = \rho_1 (\text{Re } D_x)^2 = (\text{Re } Y_1 - \text{Im } Y_1)^2, \]  

(17)

\[ \rho_{Im Z} = \frac{2}{\omega \mu} (\text{Im } Z_1)^2 = \rho_1 (\text{Im } D_x)^2 = (\text{Re } Y_1 + \text{Im } Y_1)^2, \]  

(18)

\[ \rho_{|Z|} = \frac{1}{\omega \mu} |Z_1|^2 = \frac{1}{2} \rho_1 |D_x|^2 = |Y_1|^2, \]  

(19)

\[ \phi = \arctan \frac{\text{Im } Z_1}{\text{Re } Z_1} = \arctan \frac{\text{Im } D_x}{\text{Re } D_x}, \text{ or alternatively } \phi = \frac{\pi}{4} + \arctan \frac{\text{Im } Y_1}{\text{Re } Y_1}. \]  

(20)

Many other arbitrary resistivity definitions can be made, but all of them can be derived from those given above. In fact all apparent resistivity definitions can be given as simple analytical expressions of \( S \) and \( \alpha \).

Properties of two-layer MT sounding curves

We derive the relationships between \( S \) and \( \alpha \) which correspond to points on the sounding curves where \( \rho_{app} = \rho_1 \) and \( \phi = \pi/4 \) or where \( \rho_{app} \) and \( \phi \) reach extreme values.

The condition \( \alpha > 0 \) means that \( e^a > 1 \). Since the absolute value of the reflectivity coefficient is always less than 1, it is always true that \( |S| < e^a \).

Points on the sounding curves where \( \rho_{app} = \rho_1 \)

\[ \rho_{Re Z} = \rho_1 \text{ if } \text{Re } D_x = 1 \text{ (see (10)), i.e. if } \]

\[ e^{2a} + 2Se^a \sin \alpha - S^2 = e^{2a} - 2Se^a \cos \alpha + S^2. \]  

(21)

From this equation, if \( S \neq 0 \), we have

\[ e^a (\sin \alpha + \cos \alpha) = S. \]  

(22)
\[ \rho_{\text{im},z} = \rho_1 \text{ if } \text{Im } D_x = 1 \text{ (see (11)), i.e. if} \]
\[ e^\alpha (\cos \alpha - \sin \alpha) = S, \quad (S \neq 0). \quad (23) \]
\[ \rho_{|z|} = \rho_1 \text{ if } |D_x| = 1. \text{ From (12) the condition for this is} \]
\[ \cos \alpha \cdot e^\alpha \cdot S (S^2 - e^{2\alpha}) = 0. \quad (24) \]
Excluding \( e^\alpha = 0, S = 0 \) and \( |S| = e^\alpha \), this requires that
\[ \cos \alpha = 0, \quad (25) \]
i.e. \( \lambda_1/(h-z) = 8/(1+2n), \quad (n = 0, 1, 2, 3, \ldots) \).

It should be remarked that while the \( \rho_{\text{Re},z} = \rho_1 \) and \( \rho_{\text{im},z} = \rho_1 \) crossings are functions of \( S \), \( \lambda_2 \)'s values corresponding to the \( \rho_{|z|} = \rho_1 \) crossings do not depend on \( S \), that is on \( \sigma_2 \), the conductivity of the bottom layer.

The phase \( \varphi = \frac{\pi}{4} \), when, according to its definition given in (20), the following condition is fulfilled:
\[ \varphi - \frac{\pi}{4} = \arctan \frac{-2Se^\alpha \sin \alpha}{e^{2\alpha} - S^2} = 0. \quad (26) \]

There are three possibilities:
1. \( S=0 \) means absence of conductivity contrasts;
2. \( e^\alpha = 0 \), i.e. \( \alpha \to -\infty \);
3. \( \sin \alpha = 0 \), which leads to
\[ \frac{4\pi(h-z)}{\lambda_1} = n\pi, \quad (27) \]
i.e.
\[ \lambda_1/(h-z) = 4/n, \quad (28) \]
where
\( n = 0 \) corresponds to the long-wavelength asymptote,
\( n = 1 \) corresponds to the last crossing with increasing but finite wavelength,
\( n = 2, 3, \ldots \) correspond to shorter wavelength oscillations.

The conductivity contrast does not play any role in (28). It means that the \( \varphi = \pi/4 \) crossings are independent of \( \sigma_2 \), the conductivity of the bottom. They depend only on \( \lambda_1/(h-z) \).

Sites of extreme values of the various apparent resistivity curves and of the phase curves

The sites of extreme values on different sounding curves can be given by analytical derivation with respect to \( \lambda_1/(h-z) \). Since
\[ \frac{d}{d[\lambda_1/(h-z)]} = \frac{d}{d\alpha} \cdot \frac{d\alpha}{d[\lambda_1/(h-z)]} = \frac{d}{d\alpha} \cdot \frac{-4\pi}{[\lambda_1/(h-z)]^2}, \]
the extreme values can be determined by analytical derivation with respect to \( \alpha \).
\( \rho_{ReZ} \) has extremes, if \( d\rho_{ReZ}/d\alpha = 0 \). With some algebra, the following equation can be obtained:

\[
S e^\alpha \sin \alpha (e^{2\alpha} - S^2) = 0.
\]  
(29)

The only non-trivial solution of (29) is \( \alpha = 0 \), i.e. the same as (28) corresponding to the \( \phi = \pi/4 \) crossings, with the same meaning for the various values of \( n \). The condition of extreme values on the \( \rho_{ReZ} \) sounding curve is

\( \lambda_t (h - z) = 4/n \),

(30)

where

\( n = 0 \) corresponds to infinitely long wavelengths,

\( n = 1 \) gives the extreme value corresponding to the longest finite wavelength,

\( n = 2, 3, \ldots \) correspond to the extreme values of short-wavelength oscillations.

It should be remarked again that the conditions for \( \rho_{ReZ} \) extremes are also independent of \( \rho_{2z} \), the conductivity of the bottom.

\( \rho_{ImZ} \) has extremes, if \( d\rho_{ImZ}/d\alpha = 0 \). The corresponding equation is

\[
(S^2 - e^{2\alpha}) \cos \alpha = 2S e^\alpha.
\]  
(31)

The solutions of this equation of the second degree for \( S \) are

\[ S = e^\alpha \frac{(1 \pm \sin \alpha)}{\cos \alpha}. \]

(32)

\( \rho_{1Z1} \) has extremes, if \( d\rho_{1Z1}/d\alpha = 0 \). Starting from (9), we find \( \rho_{1Z1} \) to satisfy

\[
S^3 e^\alpha (\cos \alpha - \sin \alpha) - S e^{3\alpha} (\sin \alpha + \cos \alpha) = 0,
\]  
(33)

of which the non-trivial solution is

\[ |S| = e^\alpha \sqrt{\frac{\cos \alpha + \sin \alpha}{\cos \alpha - \sin \alpha}}. \]

(34)

The phase of the impedance, \( \varphi \), has extremes, if \( d\varphi/d\alpha = 0 \). This condition is identical with \( d\tan \varphi/d\alpha \).

Starting from (20), the derivative of \( \varphi \) is zero if

\[
S^3 e^\alpha (\sin \alpha + \cos \alpha) + S e^{3\alpha} (\sin \alpha - \cos \alpha) = 0,
\]  
(35)

from which

\[ |S| = e^\alpha \sqrt{\frac{\cos \alpha - \sin \alpha}{\cos \alpha + \sin \alpha}}. \]

(36)

**Discussion**

In Table 1 all eight equations (four for the \( \rho_{app} = \rho_1 \) and \( \phi = \pi/4 \) crossings and four for the extremes) are summarized. Only two of them are known from former publications: conditions for \( \rho_{1Z1} = \rho_1 \) and \( \phi = \pi/4 \) were given (in terms of \( \sqrt{T} \)) by
Table 1. Relationship between two real parameters: the conductivity contrast $S$ and the so-called 'depth/half-skindepth' parameter $\alpha$, for several attributes of two-layer magnetotelluric sounding curves defined in several different ways.

Notation:
- layer parameters (conductivities and the layer thickness): $\sigma_1$, $\sigma_2$, and $h$. The magnetic permeability $\mu$ is constant everywhere;
- depth of observation in the first layer: $z$ (0 $\leq z \leq h$, where $z = 0$ means that the observation site is at the surface);
- angular frequency: $\omega = 2\pi/T$, where $T$ is the oscillation period;
- wavelength in the first layer: $\lambda_1 = 4\pi/\text{Re} h_1$, where the complex wave number in the first layer is $h_1 = \sqrt{\omega\mu\sigma_1}$;
- conductivity contrast (or 'reflectivity coefficient'): $S = (\sqrt{\sigma_1} - \sqrt{\sigma_2})/(\sqrt{\sigma_1} + \sqrt{\sigma_2})$;
- the 'depth/wavelength' parameter: $\alpha = 4\pi(h - z)/\lambda_1$;
- $\rho_{\text{Re} Z}$, $\rho_{\text{Im} Z}$, $\rho_{|Z|}$ are apparent resistivities based on different parts of the impedance $Z$ indicated by indices; $\varphi$ is the magnetotelluric phase.

| $\rho_{\text{Re} Z} = 1/\sigma_1$, if | $\rho_{\text{Im} Z} = 1/\sigma_1$, if | $\rho_{|Z|} = 1/\sigma_1$, if | $\varphi = \pi/4$, if |
|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| Locations where $\rho_{\text{eq}} = \rho_1$ or $\varphi = \pi/4$. | $e^\varphi (\sin \alpha + \cos \alpha) = S$ | $e^\varphi (\cos \alpha - \sin \alpha) = S$ | (independently of $S$): $\frac{4\pi}{\alpha} = \frac{\lambda_1}{h - z} = \frac{8}{1 + 2n}$, where $n = 0, 1, 2, \ldots$ |
| $\rho_{\text{Re} Z}$ has extremes, if (independently of $S$) | $\rho_{\text{Im} Z}$ has extremes, if (independently of $S$) | $\rho_{|Z|}$ has extremes, if (independently of $S$) | $\varphi$ has extremes, if |
| $\frac{4\pi}{\alpha} = \frac{\lambda_1}{h - z} = \frac{4}{n}$, where $n = 1, 2, 3, \ldots$ | $e^\varphi (1 \pm \sin \alpha)/\cos \alpha = S$ | $e^\varphi \sqrt{\cos \alpha + \sin \alpha}/\cos \alpha = |S|$ | $e^\varphi \sqrt{\cos \alpha - \sin \alpha}/\cos \alpha = |S|$ |
Figure 1. Real and imaginary parts of the dimensionless magnetotelluric impedance $D_z$ over a two-layer model having a conductivity ratio $\sigma_2/\sigma_1 = 100$ in the complex plane. The numbers refer to $\lambda_1/(h-z)$ values corresponding to $\text{Re} D_z = 1$, $\text{Im} D_z = 1$ and $\varphi = \pi/4$, and also to $\lambda_1/(h-z)$ values corresponding to extremes of $\text{Re} D_z$, $\text{Im} D_z$, $|D_z|$ and $\arctan \text{Im} D_z/\text{Re} D_z$. Their positions on the corresponding resistivity and phase curves are shown in Fig. 2.

Cagniard (1953). The observation sites should not necessarily be on the surface: they can be situated anywhere at depth $z$ in the first layer.

A new result obtained here is that the $\rho_{\text{Re} Z}$ extremes occur at the same $\lambda_1/(h-z)$ values as the $\varphi = \pi/4$ crossings. It is also interesting that these three properties are independent of the conductivity contrast (i.e. of the conductivity of the bottom), whereas in the remaining five equations given in Table 1, $S$ also appears in the trigonometric-exponential equations in such a way that $S$ (or $|S|$) can always be expressed directly in terms of $\alpha$.

In Fig. 1, the dimensionless impedance elements (defined in (10) and (11)) for a two-layer half-space, with $\sigma_1 = 100\sigma_2$, are plotted in the complex plane. The numbers along the curve refer to $\lambda_1/(h-z)$ values defined in Table 1. In Fig. 2, the three apparent resistivity sounding curves and the phase curve are each plotted as a function of $\lambda_1/(h-z)$. The $\rho_{\text{app}} = \rho_1$ and $\varphi = \pi/4$ crossing and the extremes are denoted on each curve by stars. They appear at the $\lambda_1/(h-z)$ values shown in Fig. 1. (Passages and extremes corresponding to the small short-period oscillations ($\lambda_1/(h-z)$ values smaller than 2.29) are not considered in these two figures.)

Conclusion

The simplicity of the equations in Table 1 indicates that $S$ and $\alpha$ are physically
meaningful key parameters in magnetotellurics. Perhaps more attention should be paid to formulating magnetotelluric problems in terms of $S$ and $\alpha$. The question of how to use, in magnetotelluric interpretations, the reflectivity coefficient and the ‘depth/half-skinc depth’ ratio is possibly worth studying anew.

One possibility is as follows. For the unknowns, $S$ and $\alpha$, eight different conditions, shown in Table 1, can be considered. From these conditions $(h - z)$ and $S$ can be determined in several different ways. The set of equations given here thereby provides a means of deriving the depth and/or the reflectivity from the very early phases (that is from the short-period range) of the sounding curves. Of course for such a method to be useful, reasonably noise-free data are necessary. The prospects of this method seem particularly useful when using the sounding curves $\rho_{ReZ}$ and $\varphi_j$ since at a given period these give information from greater depths than the other curves (Szarka and Fischer 1989; Szarka et al. 1994).

The system of passages and extremes of different sounding curves also appears to be applicable to the interpretation of precise near-surface measurements. For the practical applicability of our results, further studies on $S$ and $\alpha$ in more complicated
situations would appear to be necessary. In any case, the author encourages MT analysis to be considered more in terms of reflectivity and the depth/half-skindepth ratio, as it has been done recently by Nagy (1996).

Unfortunately the concept of wavelength \( \lambda_1 \) (although it is still used by the Russians) has not been fully accepted in EM induction work in Western Europe and North America. For example, four years ago the majority of the electromagnetic working group of IAGA voted to plot the MT sounding curves as a function of the logarithm of the period \( T \) (Hobbs 1992). Since \( \sqrt{T} \) and not \( T \) has a linear relationship with the wavelength \( \lambda_1 \), this vote further contributed to discrediting the 'wavelength' concept.

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Appendix

Derivation of basic MT relations in the two-layer cases

Faraday's and Ampère's laws as used in 1D MT are as follows:

\[
\text{rot } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{A1}
\]

\[
\text{rot } \mathbf{H} = \sigma \mathbf{E}, \tag{A2}
\]

where \( \mathbf{E} \) is the electric field, \( \mathbf{B} \) is the magnetic induction vector and \( \mathbf{H} \) is the magnetic field (\( \mathbf{B} = \mu \mathbf{H} \), where \( \mu \), the scalar constant of magnetic permeability, is assumed). The conductivity \( \sigma \) indicates a homogeneous and isotropic space.

The differential equation for \( \mathbf{E} \) is

\[
\Delta \mathbf{E} - \sigma \mathbf{E} = 0. \tag{A3}
\]

Assuming an 'i\( \omega \)' time dependence (\( t \) is time and \( \omega \) is the angular frequency), \( \mathbf{E} = (E_x(x) \cdot e^{i\omega t}, 0, 0) \) and if \( \partial / \partial x = \partial / \partial y = 0 \), then

\[
\frac{d^2 E_x}{dx^2} - \omega^2 \mu \sigma E_x = 0. \tag{A4}
\]

The general solutions of this differential equation of type, \( \ddot{x} - k^2 x = 0 \), are

\[
E_x = A e^{-kx} + B e^{kx}, \quad \text{where } k^2 = i\omega \mu \sigma. \tag{A5}
\]

\( \mathbf{H} = (0, H_y(x) \cdot e^{i\omega t}, 0) \) is given by the 1D version of Faraday's law,

\[
H_y = \frac{-1}{i\omega \mu} \frac{dE_x}{dx}. \tag{A6}
\]
In the two-layer case (with conductivities \( \sigma_1 \), \( \sigma_2 \) and layer thickness \( h \)), in the first and the second layers the field components can be expressed as follows:

\[
E_{x1} = A_1 e^{-h_1 x} + B_1 e^{h_1 x}, \quad E_{x2} = A_2 e^{-h_2 x} + B_2 e^{h_2 x},
\]
\[
H_{n1} = \frac{k_1}{\omega \mu} (A_1 e^{-h_1 x} - B_1 e^{h_1 x}), \quad H_{n2} = \frac{k_2}{\omega \mu} (A_2 e^{-h_2 x} - B_2 e^{h_2 x}). \tag{A7}
\]

The four unknowns, \( A_1 \), \( B_1 \), \( A_2 \) and \( B_2 \), can be determined from four conditions, which can be expressed as follows:

1. Due to the absence of upward waves in the second layer, \( B_2 = 0 \).
2. Boundary conditions for \( E_x \): \( E_x(z = h) = E_x(z = 0) \).
3. Boundary conditions for \( H_n \): \( H_n(z = h) = H_n(z = 0) \).
4. The fourth condition is that the amplitude of the surface field \( H_0 \) is arbitrary, i.e. \( H_0(z = 0) = H_0 = 1 \).

The three equations for \( A_1, B_1 \) and \( A_2 \) are then

\[
A_1 e^{-h_1 x} + B_1 e^{h_1 x} = A_2 e^{-h_2 x}, \tag{A8}
\]
\[
k_1 A_1 e^{-h_1 x} - k_1 B_1 e^{h_1 x} = k_2 A_2 e^{-h_2 x}, \tag{A9}
\]
\[
\frac{i \omega \mu}{k_1} = A_1 - B_1, \tag{A10}
\]

with the solutions for \( A_1, A_2 \) and \( B_1 \) being

\[
A_1 = \frac{i \omega \mu}{k_1} \frac{e^{2h_1 x} - k_1 - k_2}{e^{2h_1 x} - k_1 + k_2}, \quad A_2 = \frac{2i \omega \mu}{k_1 + k_2} \frac{e^{(k_1 + k_2)h} - k_1 - k_2}{e^{2h_1 x} - k_1 + k_2}, \tag{A11}
\]

\[
B_1 = \frac{i \omega \mu}{k_1} \frac{k_1 + k_2}{e^{2h_1 x} - k_1 + k_2}, \quad (B_2 = 0).
\]

The field components become

\[
E_{x1} = \frac{i \omega \mu}{k_1} \frac{e^{h_1 x} + k_1 - k_2}{e^{2h_1 x} - k_1 + k_2}, \quad E_{x2} = \frac{2i \omega \mu}{k_1 + k_2} \frac{e^{h_2 x} - e^{(k_1 + k_2)h}}{e^{2h_1 x} - k_1 - k_2}, \tag{A12}
\]
\[
H_{n1} = \frac{e^{h_1 x}}{e^{2h_1 x} - k_1 + k_2}, \quad H_{n2} = \frac{2k_2}{k_1 + k_2} \frac{e^{h_2 x}}{e^{2h_1 x} - k_1 - k_2}.
\]
With \( k_j = \sqrt{\mu_j / \rho_j} \) (\( j = 1, 2 \)), the impedances \( Z_1 \) and \( Z_2 \) in media 1 and 2 are

\[
Z_1 = \frac{E_{z1}}{H_{z1}} = \frac{i \omega \mu}{k_1} \cdot \frac{e^{2b_1(h-s)} + \frac{k_1 - k_2}{k_1 + k_2}}{e^{2b_1(h-s)} - \frac{k_1 - k_2}{k_1 + k_2}} = \sqrt{\frac{i \omega \mu \sqrt{\rho_1}}{k_1 + k_2}} \cdot \frac{e^{2b_1(h-s)} + \frac{k_1 - k_2}{k_1 + k_2}}{e^{2b_1(h-s)} - \frac{k_1 - k_2}{k_1 + k_2}}.
\]

(A13)

\[
Z_2 = \frac{E_{z2}}{H_{z2}} = \frac{i \omega \mu}{k_2} = \sqrt{\frac{i \omega \mu \sqrt{\rho_2}}{k_2}}.
\]

(A14)

References


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