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SLINGRAM MEASUREMENTS IN THE MECSEK MOUNTAINS, HUNGARY

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The so-called Slingram measurements form a family of frequency-domain electromagnetic methods, when the transmitter is a vertical magnetic dipole, and the vertical component of the magnetic field is measured, at some distance from the transmitter. The measured value is usually the imaginary (out-of-phase) part of the vertical magnetic field component. Such an equipment was manufactured at the University of Neuchâtel (Switzerland) and supplied to the Geodetic and Geophysical Research Institute in frame of a Swiss-Hungarian co-operation, supported by the Swiss National Research Foundation. As a part of test measurements, Slingram profilings and soundings were carried out for exploration of karstic water reservoirs connected to tectonically weak zones in the Mecsek Mountains, close to Pécs (Hungary). One-dimensional inversion and comparison with other geo-electromagnetic techniques as radio-magnetotellurics (RMT) and radiofrequency-electromagnetics (RF-EM) resulted in good and interpretable results.

Keywords: electromagnetic methods; karstic water reservoir; Mecsek Mountains; Slingram measurement

Introduction

The so-called Slingram measurements form a family of frequency-domain electromagnetic methods, when the transmitter is a vertical magnetic dipole, and the vertical component of the magnetic field is measured, at some distance from the transmitter. The measured value is usually the imaginary (out-of-phase) part of the vertical magnetic field component, but the real (in-phase) part, containing the primary field (that is the magnetic field in full free space) is also measured.

There are a lot of variants of Slingram measurements, e.g. airborne electromagnetic measurements, the so-called EM31 measurements, and the Apex MinMax technique. An equipment using the Slingram principle has been developed at the Hydrogeological Centre of the Neuchâtel University. The transmitter-receiver separation varies between 7.07 m – 56.56 m, in the operating frequency varies between 220 Hz and 14080 Hz.

The measured data themselves depend not only on the resistivity of the medium at depth, but they depend also on other parameters: the transmitter-receiver separation, the frequency, and the momentum of the transmitter. In order to remove

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the effects of these latter parameters, the measured data are usually transformed into apparent resistivities. In Slingram measurements the apparent resistivity is usually computed from the imaginary part of the magnetic field, but it could be equally calculated from other components: from the real part, from the absolute value, and even from the phase. The apparent resistivity computation had a greater significance decades ago, when the data processing had a limited possibility. Nowadays, with the advent of fast computers, layer parameters can be directly determined from the response function. The inverted parameters (depths and layer resistivities) give of course more information about the subsurface structure, than the apparent resistivity itself.

In this paper the results of Slingram test measurements, carried out in the Mecsek Mountains (Southern Transdanubia, Hungary) are shown, where near- surface structural changes in connection of possible karstic water-reservoirs were studied.

In the first part of the paper the theoretical aspects of the Slingram modelling and inversion are summarised, then the field results are presented.

Mathematical modelling of Slingram measurements for layered earth

A widely used data processing method in geophysics is the one-dimensional inversion. It means, that an algorithm searches for layer parameters (resistivities and thicknesses), having the same electromagnetic response, which was measured in the field. The inversion algorithm needs the solution of the direct problem. Let us review briefly the basic formulae. The vertical and horizontal components of the electromagnetic field due to a vertical magnetic dipole is given by a Hankel transform:

$$
\begin{aligned}
H_z(\omega) &= \frac{IS}{4\pi}\left\{\int_0^\infty J_0(\lambda r)\lambda^2 e^{-\lambda(z+h)}R_0(\lambda)d\lambda - \frac{1}{R^3} + \frac{3(z-h)^2}{R^5}\right\} \\
H_r(\omega) &= \frac{IS}{4\pi}\left\{\int_0^\infty J_1(\lambda r)\lambda^2 e^{-\lambda(z+h)}R_0(\lambda)d\lambda + \frac{3r(z-h)}{R^5}\right\}
\end{aligned}
\tag{1}
$$

where IS is the momentum of the dipole, $R=\sqrt{r^2+(z-h)^2}$, $\omega=2\pi f$, and $R_0(\lambda)$ is the kernel function depending on the layer parameters, given by the recursion formula:

$$
R_j(\lambda) = \frac{\dfrac{\lambda_j-\lambda_{j+1}}{\lambda_j+\lambda_{j+1}} + R_{j+1}(\lambda)e^{-2\lambda_{j+1}d_{j+1}}}{1+\dfrac{\lambda_j-\lambda_{j+1}}{\lambda_j+\lambda_{j+1}}R_{j+1}(\lambda)e^{-2\lambda_{j+1}d_{j+1}}}
$$

$\lambda_j = \sqrt{\lambda^2+i\omega\mu\sigma_j}$, h, z are the height of the transmitter, and of the receiver correspondingly (in our case, if both the transmitter and the receiver are at the surface of a horizontal half-space $h=z=0$), σ_j is the conductivity of the j-th layer, d_j is the thickness of the j-th layer, J_0, J_1 are Bessel functions.

In Slingram measurements only the vertical component of the electromagnetic field is measured. However, when the plane of the receiver coil is not parallel to the surface, the measured value is effected also by the horizontal component of the magnetic field.

In the case when the plane of the transmitter coil is not parallel to the surface, the momentum of the transmitter can be separated into vertical and horizontal components, therefore the magnetic field due to a horizontal magnetic dipole is also necessary to be taken into account.

The vertical component of the magnetic field due to a horizontal magnetic dipole:

$$H_z(\omega) = \frac{IS}{4\pi}\left\{ -\int_0^\infty J_1(\lambda r)\lambda^2 e^{-\lambda(z+h)} R_0(\lambda) d\lambda + \frac{3(z-h)r}{R^5} \right\} .$$

The horizontal component of the magnetic field:

$$\begin{aligned} H_r(\omega) &= \frac{IS}{4\pi}\left\{ -\frac{1}{r}\int_0^\infty J_1(\lambda r)\lambda e^{-\lambda(z+h)} R_0(\lambda) d\lambda \right\} + \\ & \frac{IS}{4\pi}\left\{ \int_0^\infty J_0(\lambda r)\lambda^2 e^{-\lambda(z+h)} R_0(\lambda) d\lambda - \frac{2}{R^3} \right\} . \end{aligned} \qquad (2)$$

In case of tilted transmitter and receiver, at the transmitter side both the horizontal and vertical component due to the transmitter dipole has to be taken into account, and at the receiver side both the vertical and horizontal components of the magnetic field should be computed. With a tilted system the measured value is given by the formula:

$$\begin{aligned} H_z^{(0)}(\omega) &= (H_z^{(v)}(\omega)\cos\Theta - H_z^{(h)}(\omega)\sin\Theta)\cos\phi - \\ & -(H_r^{(v)}(\omega)\cos\Theta - H_r^{(h)}(\omega)\sin\Theta)\sin\phi\,, \end{aligned}$$

where $H_z^{(0)}$ is the measured field component, $H_z^{(v)}$, $H_r^{(v)}$ are the magnetic field components due to the vertical magnetic dipole, $H_z^{(h)}$, $H_r^{(h)}$ are the magnetic field components due to the horizontal magnetic dipole, and Θ, ϕ are the tilt angle of the transmitter and the receiver, correspondingly.

If the misorientation angles Θ and ϕ were exactly known, there would be no problem to carry out an exact one-dimensional modelling and inversion. If the misorientation angles are not known, in the one-dimensional inversion Θ and ϕ angles should be considered as unknowns. In this latter case the inversion results are not only the layer parameters, but also the two tilt angle values. This kind of inversion is possible to be done in an appropriate induction number domain, with sufficient number of frequencies.

Apparent resistivity computation

For small induction numbers, the apparent resistivity can be computed simply from the imaginary part of the magnetic field. If the induction number $Q = r^2\mu\sigma\omega$ is small (much smaller than 1), then the recursion formula, which determines the kernel function in formulae (1) and (2) becomes simpler:

$$R_j(\lambda) \approx \frac{i\omega\mu(\sigma_j - \sigma_{j+1})}{4\lambda^2} + R_{j+1}(\lambda)e^{-2\lambda d_j} \, .$$

Then $\mathrm{Im}H_z$ is obtained as follows:

$$\begin{aligned} \mathrm{Im}H_z &= -\frac{IS}{4\pi}\left(\frac{\sigma_1\omega\mu}{4r} + \frac{(\sigma_2-\sigma_1)\omega\mu}{4\sqrt{r^2+4d_1^2}} + \frac{(\sigma_3-\sigma_2)\omega\mu}{4\sqrt{r^2+4(d_1+d_2)^2}} + \dots \right. \\ &\left. + \frac{(\sigma_n-\sigma_{n-1})\omega\mu}{4\sqrt{r^2+4(d_1+d_2+\ldots+d_{n-1})}}\right) . \end{aligned} \tag{3}$$

The kernel function $R_0(\lambda)$ for homogeneous half-spaces is as follows:

$$R_0(\lambda) = \frac{\lambda - \lambda_1}{\lambda + \lambda_1} \approx \frac{i\omega\mu\sigma_1}{4\lambda^2} \, .$$

From the kernel function the vertical component of the magnetic field over a homogeneous half-space with conductivity σ_1 is obtained as by the following formula:

$$\mathrm{Im}H_z \approx -\frac{IS}{4\pi}\frac{\omega\mu\sigma_1}{4\sqrt{r^2+(z+h)^2}} \, .$$

Then the σ_1 conductivity can be expressed as a function of $\mathrm{Im}H_z$. In this way over a more complicated structure the measured $\mathrm{Im}H_z$ can be transformed into apparent conductivity as follows:

$$\sigma_a = \frac{4\mathrm{Im}H_z}{H_z^{(p)}\omega\mu r^2} \, , \tag{4}$$

where $H_z^{(p)}$ the primary magnetic field, and both the transmitter and the receiver are at the surface $(z = 0,\ h = 0)$. The reciprocal value of the apparent conductivity is the apparent resistivity:

$$\varrho_a = \frac{H_z^{(p)}\omega\mu r^2}{4\mathrm{Im}H_z} \, .$$

After substituting formula (3) into formula (4) we obtain a very informative form for the apparent conductivity over a layered half-space:

$$\begin{aligned} \sigma_a &= \sigma_1 - \sigma_1 P(z_1) + \sigma_2\left(P(z_1) - P(z_2)\right) + \dots \\ &\dots + \sigma_{n=1}\left(P(z_{n-2}) - P(z_{n-1})\right) + \sigma_n P(z_{n-1}) \, , \end{aligned} \tag{5}$$

where

$$P(z_i) = \frac{1}{\sqrt{\left(\frac{2z_i}{r}\right)^2 + 1}}, \qquad z_i = \sum_{j=1}^{i} d_j .$$

It should be observed for small induction numbers that the apparent conductivity (and also its reciprocal value, the apparent resistivity) is independent of the frequency. It depends only on the layer parameters and on the transmitter-receiver separation. In this case the penetration depth of the Slingram measurement is independent of the frequency. If we want to obtain information from different depths at a measuring site, we need to change the transmitter-receiver separation.

One-dimensional inversion

The goal of electromagnetic geophysics is to determine the conductivity distribution of medium below the surface. When the conductivity does not change in horizontal direction, that is the model is layered, a so called one-dimensional inversion can be performed. Its results are the layer parameters: resistivities and thicknesses. In the inversion it is assumed that for small variations in the layer parameters, the observed data are linear functions of the layer parameters. Shortly let us look over the theoretical aspects of the inversion.

The components of the vector $\mathbf{m}$ are the $\mathrm{Re}H_z$, or $\mathrm{Im}H_z$ values, measured at a measuring site, using different frequencies or/and different transmitter-receiver separations. The components of the vector $\mathbf{p}$ are the layer parameters. The inversion is carried out in several iteration steps. Let us assume that the $\mathbf{p}_i$ estimation of the parameter vector is known. The data vector belonging to it is $\mathbf{m}_i$. $\mathbf{m}_0$ means the measured data. Let $\Delta\mathbf{m} = \mathbf{m}_0 - \mathbf{m}_i$, and $\mathbf{p}_{i+1} = \mathbf{p}_i + \Delta\mathbf{p}$. The parameter vector must be modified in such a way, that the resulting $\mathbf{m}_{i+1}$ data vector belonging to the parameter vector gets closer to $\mathbf{m}_0$. The vector $\Delta\mathbf{p}$ can be computed solving the equation

$$\Delta\mathbf{m} = \mathbf{J}\Delta\mathbf{p} . \tag{6}$$

The $\mathbf{J}$ Jacobian matrix contains partial derivatives, which are known from the direct calculation. $\mathbf{m}_i$ is known from $\mathbf{p}_i$. Since $\mathbf{m}_0$ is measured, $\Delta\mathbf{m}$ is known, too. If matrix $\mathbf{J}$ in Eq. (6) were an invertable square matrix, then we could easily obtain the unique solution. Unfortunately in most cases $\mathbf{J}$ is not an invertable square matrix. A generalized inverse is based on the singular value decomposition (SVD) of the Jacobian matrix $\mathbf{J}$ (Jackson 1972):

$$\mathbf{J} = \mathbf{U}\Lambda\mathbf{V}^T, \quad \mathbf{U}^T\mathbf{U} = \mathbf{I}, \quad \mathbf{V}^T\mathbf{V} = \mathbf{I} , \tag{7}$$

where $\mathbf{I}$ is the identity matrix, and the Λ is a diagonal matrix containing the eigenvalues:

$$\Lambda = \begin{pmatrix} \lambda_1 & 0 & 0 & \dots & 0 \\ 0 & \lambda_2 & 0 & \dots & 0 \\ 0 & 0 & \lambda_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \lambda_n \end{pmatrix} .$$

The generalized inverse is obtained as follows:

$$\mathbf{J}^{+} = \mathbf{V}\Lambda^{-1}\mathbf{U}^{T} .$$

In the knowledge of $\Delta\mathbf{m}$, $\Delta\mathbf{p}$ is to be computed with the following formula:

$$\mathbf{V}\mathbf{V}^{T}\Delta\mathbf{p} = \mathbf{J}^{+}\mathbf{U}\mathbf{U}^{T}\Delta\mathbf{m} ,$$

where $\mathbf{R} = \mathbf{V}\mathbf{V}^{T}$ is the resolution matrix, $\mathbf{S} = \mathbf{U}\mathbf{U}^{T}$ is the information density matrix.

Inasmuch the resolution matrix is the identity matrix, each element of $\Delta\mathbf{p}$ is individually given, otherwise only some linear combination of the elements can be determined. E.g. for the layer parameters it means that only the product of the thickness and the conductivity of a thin conductive layer can be given. After solving Eq. (6), a sum of $\mathbf{p}_i$ and $\Delta\mathbf{p}$ provides a new estimation for the layer parameters. Then, with the modified layer parameters, a new forward computation is performed, resulting in a new iteration step. The iteration process stops, when either the norm of $\Delta\mathbf{m}$ is small, or $\mathbf{p}_i$ and $\mathbf{p}_{i+1}$ do not differ any more from each other. The results are reliable, if the number of the measured data (size of $\mathbf{m}$) exceeds the number of layer parameters (size of $\mathbf{p}$), and the initial model is not very far from the reality.

The multi-dimensional case

It is not guaranteed that the medium is a one-dimensional one. There are already two- and three-dimensional inversion algorithms, working on a similar principle as the one-dimensional inversion does. The main difficulty with the multi-dimensional inversions is, that they are rather time-consuming, even if the fastest computers are used. The reliability of the obtained model is often questionable, too. Despite of these problems, in some special cases they may give reasonable results. A plot of the measurements along a profile, together with some additional geological information tells us, whether the structure below the surface is two- or three-dimensional. In order to tell, whether the source of the anomaly is a special multi-dimensional structure, forward computations are used. For example the electromagnetic field due a conductive thin plate is known. The fracture zones at depths may be modelled with the plate model. In this case it is already possible to carry out an inversion with some limitations, of course. It can be determined e.g., if the anomaly observed along a profile is equivalent with the effect of a plate at a given depth with a given tilt angle and conductance.

Case histories: Measurements in the Mecsek Mountains

In the area of Mecsek Mts. Slingram profilings and soundings have been used for the exploration of the karstic water reservoirs connected to the tectonically weak zones (fracture, faults, etc.).

The Slingram instrument we used was manufactured at the Neuchâtel University (Hydrogeological Centre) and supplied to the Geodetic and Geophysical Research Institute in the frame of a grant given by the Swiss National Science Foundation. The measurements have been carried out with vertical magnetic dipole in the field at 4 frequencies (440, 1760, 7040 and 14080 Hz) with constant distance of 40 m between the transmitter and receiver loops.

Besides the Slingram measurements other electromagnetic techniques: RF-EM and RMT radiofrequency methods were also applied. These measurements enable us to reduce the ambiguity of the data.

Case history A

The main karstic water source in the Mecsek Mts. is the so-called "Tettye spring" in Triassic limestone. This cannot supply enough water for the town Pécs, therefore expensive water transport is needed from the Danube region. Secondary water resources in the Mecsek Mts. could decrease these high expenses. It is suspected that the N-S valleys running from the mountains towards the town could also represent karstic water reservoirs as the water springs on their southern ends point to that direction. Nevertheless, the water close to the town is already contaminated by inhabitants living around the valleys, having neither tap-water nor canalization. Therefore the aim of the measurements with Slingram and other techniques was to find water reservoirs at the highest points of the valleys where the karstic water is not yet contaminated.

The karstic water reservoirs in the limestone have lower electric resistivity than the undisturbed (fresh) limestone, therefore they can be followed by geoelectric methods, among them by Slingram and by the radiofrequency methods, too. The Slingram profilings have been carried out through the valleys which were crossed, following the tourist paths.

In Fig. 1a the out-of-phase components along the profile A are shown having two definite decreases in amplitude, corresponding to smooth topographical deepenings (valleys). The anomalies appear first of all at the highest frequencies (7040 and 14080 Hz), referring to the close-to-surface character of the resistivity change. The same tendency was recognized in the inphase components (Fig. 1b). It is to be noted that at the end of the profile there is a strong increase in both components, probably due to artificial sources (water tubes or electric lines) in the soil.

As a result of one-dimensional inversion of this dataset 2 or 3 layers could be determined. The fitting between the measured and the modelled data is illustrated in Figs 2a and b, showing a good (Fig. 2a) and a relatively bad fitting (Fig. 2b). The fitting is generally much better in case of the out-of-phase components.

The layer sequence along the profile A is given in Fig. 3. The great increase

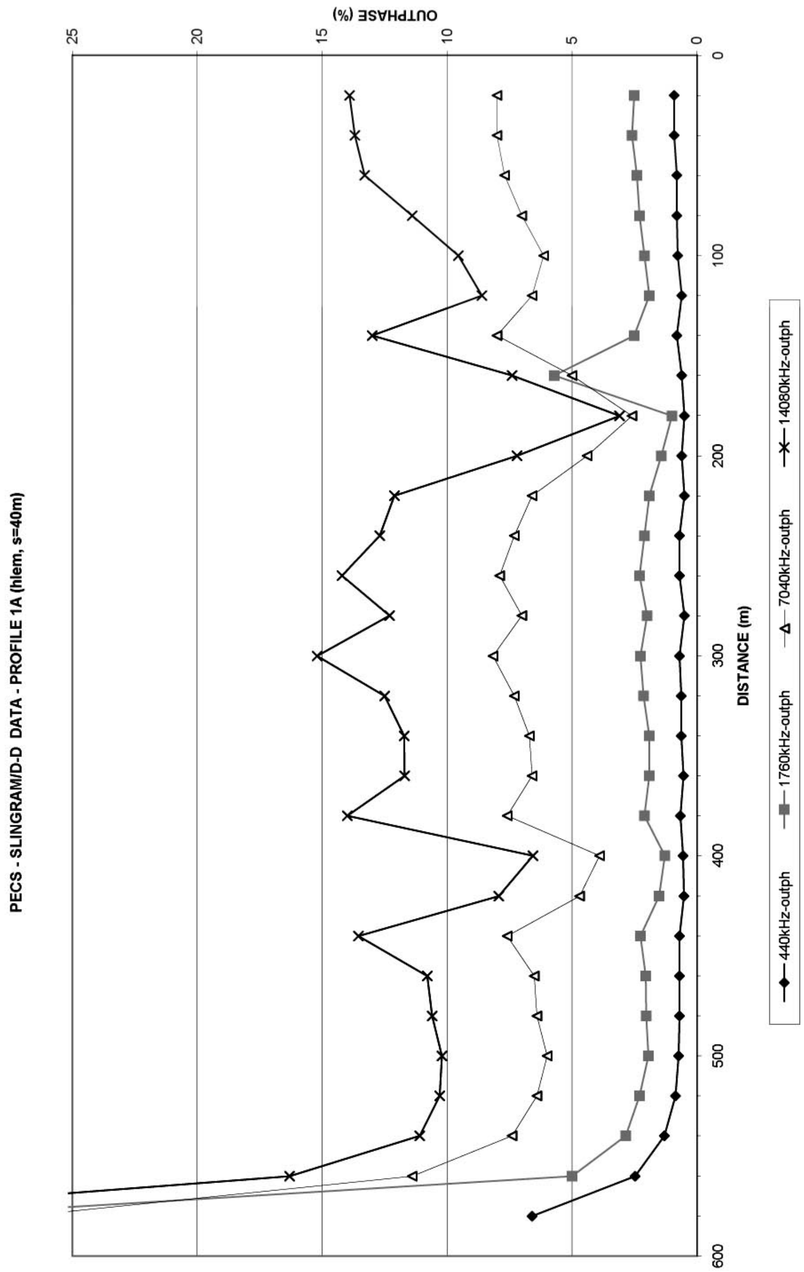

Fig. 1a. Slingram profiles along profile A. Out-of-phase data

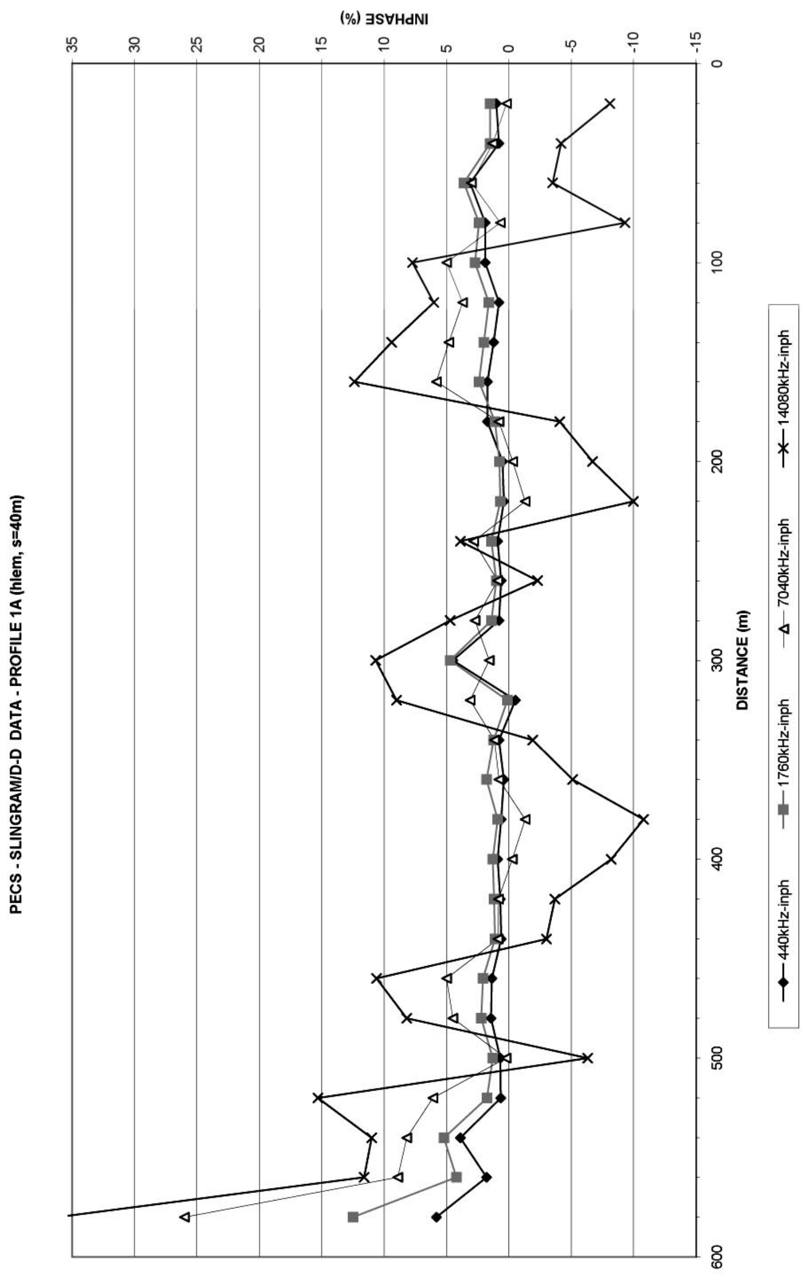

Fig. 1b. Slingram profiles along profile A. Inphase data

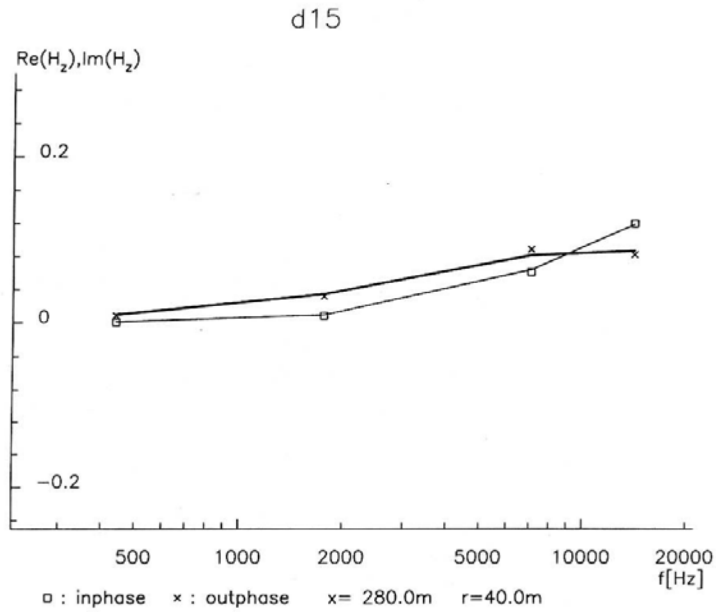

Fig. 2a. Characteristic misfits between the observed and inverted data. An example for a good fitting

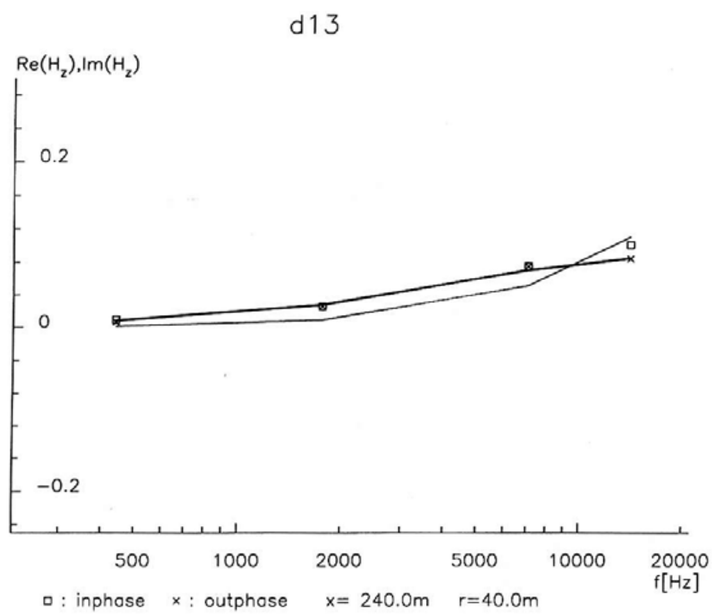

Fig. 2b. Characteristic misfits between the observed and inverted data. An example for a bad fitting

in the out-of-phase component between x = 180 and 260 corresponds to a strong conductive zone with thickness greater than about 50 m under a resistive cover. Along the whole profile an about 10 m thick conductive layer could be determined at a depth of about 10–15 m, which thickens between x = 450 and 500 m. These conductive zones could be perspective for water supply.

Case history B

Along another profile having zero misorientation effect due to a favourable topography very curious Slingram and other electromagnetic data were obtained. This profile crosses very different rock ensembles with water spring at their contact. As it is shown in Fig. 4a, the out-of-phase components have positive and negative peaks. The most interesting anomaly appears before the so called "black rock spring" with a very high negative peak, which — due to its negative sign — cannot be transformed into resistivity values.

The negative peak in the out-of-phase components is followed by a positive increase, which corresponds to a resistivity decrease. This is certainly due to water saturated rocks as indicated by the water outflow at the black rock spring. There are a few consecutive similar phenomena along this profile, hinting at the very complex geological structure of the area. The character of the inphase components is quite the same (as shown in Fig. 4b), having peaks at the same sites, where the out-of-phase components do. The amplitudes in both components reach here also their greatest values at higher frequencies hinting again at the close-to-surface character of the subsurface structure A. 1D inversion of these data resulted in the layer profile shown in Fig. 5. These zones have been found around sites x = 100, 225 m and at the end of the profile near the black rock spring (x = 400 m). Their thickness can reach 40–50 m.

Figure 6 shows a comparison between Sligram out-of-phase values and the RF-EM (216 kHz) out-of-phase data. Although the frequency is much higher in the latter case, the character of the indications is very similar. The only exception is observed around the black-rock spring from where the RF-EM curve becomes very flat. Its relative amplitudes are almost the same as those of the Slingram measurements. The RMT data are plotted together with RF-EM out-of-phase values in Fig. 7. The aforementioned flat character of the RF-EM out-of-phases at the black rock spring similarly appears in the RMT resistivity data, nevertheless, after their long and continuous decrease, starting before the "RÁBAI-fa". This means that the RMT resistivity also reaches its lowest value at the black rock spring corresponding to the possible high water saturation of the rocks.

Conclusion

The Slingram profiling/sounding can indicate those near-surface structural changes which are in connection with water reservoir where the rocks are saturated by water. Therefore this method can be applied to explore water reservoirs in karstic area of the Mecsek Mts. in Triassic limestone, too.

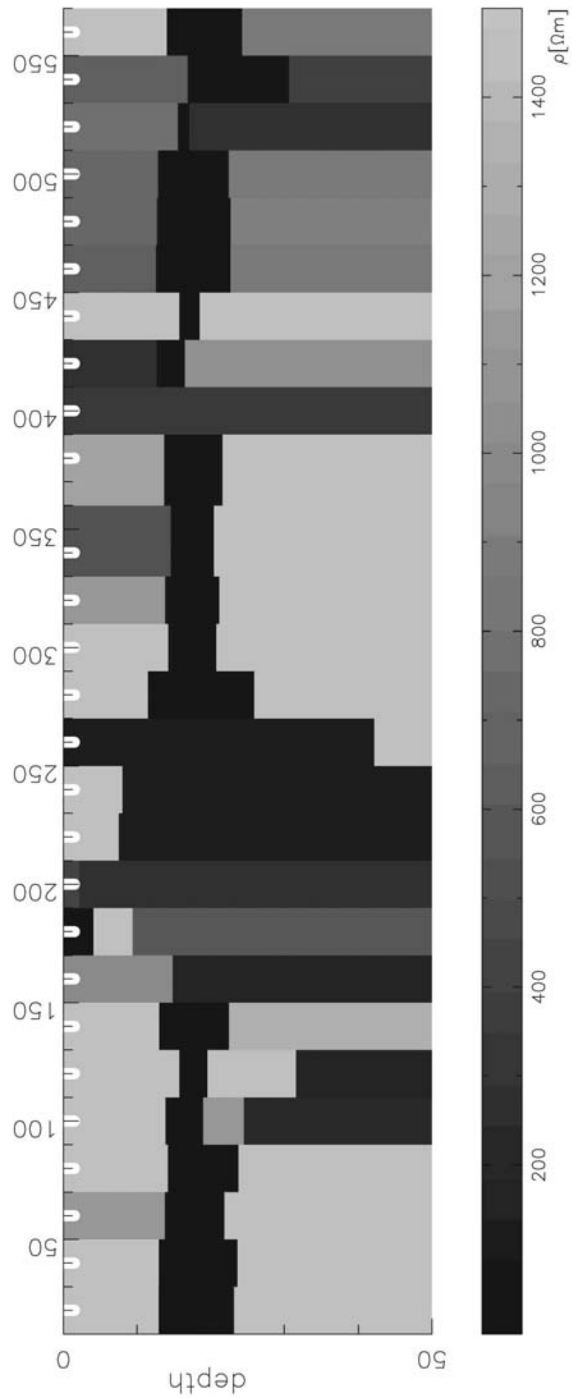

Fig. 3. Inverted resistivity section along profile A as obtained from one-dimensional inversion

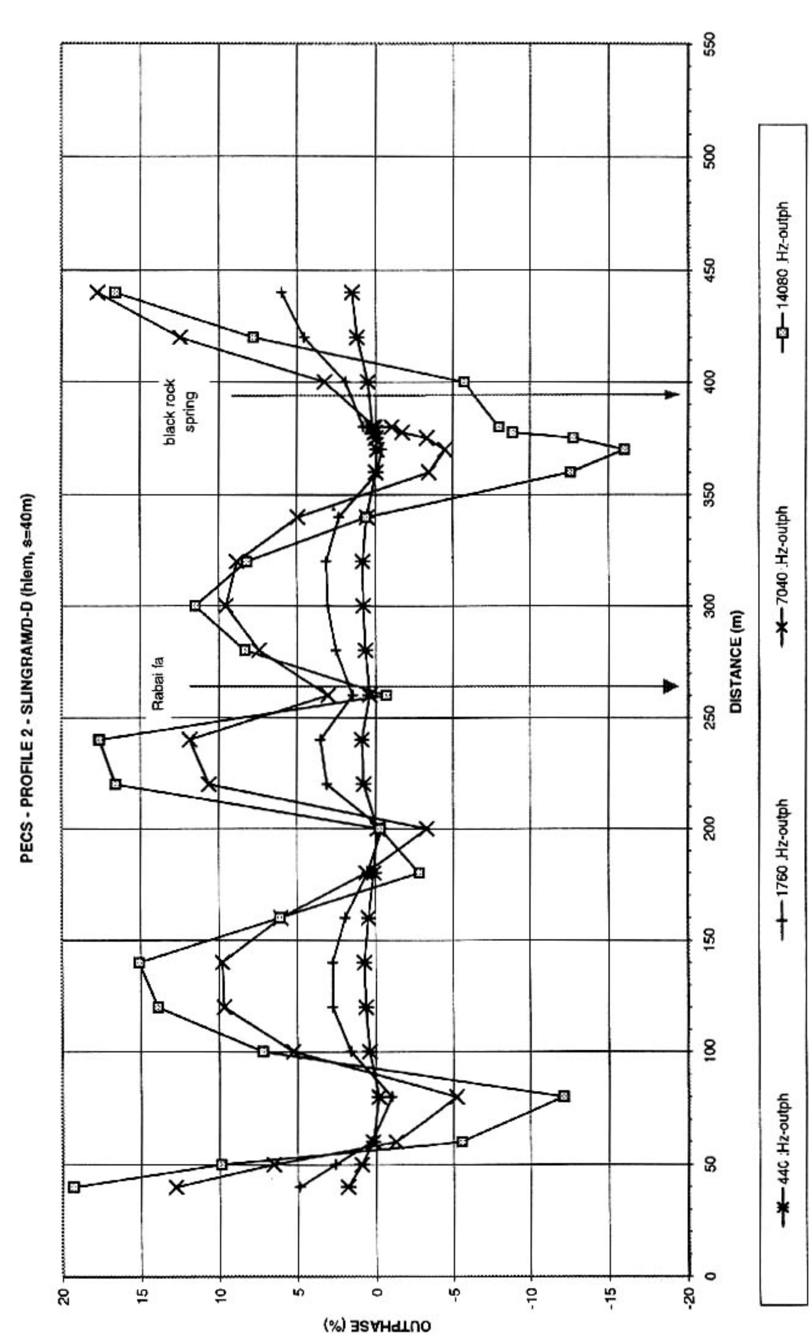

Fig. 4a. Slingram profiles along profile B (at the Rabai fa). Out-of-phase data

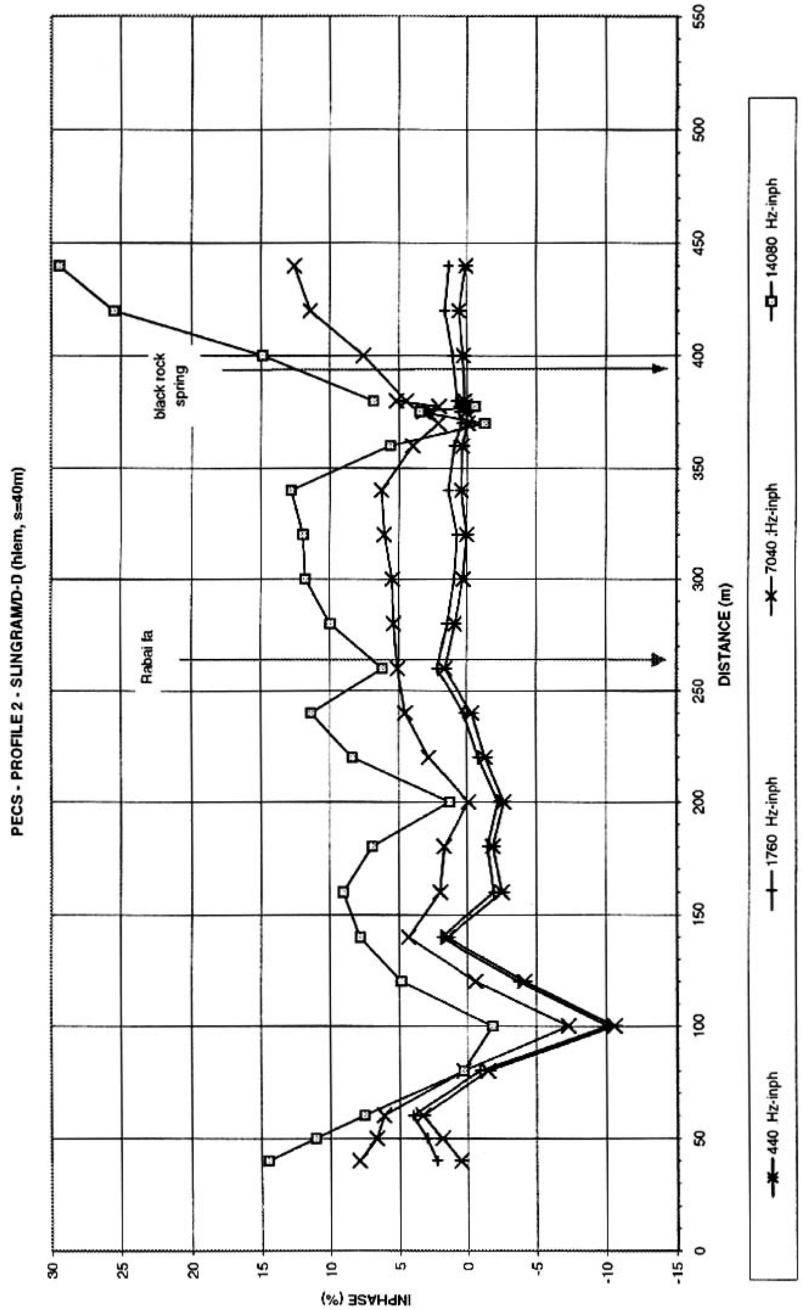

Fig. 4b. Slingram profiles along profile B (at the Rabai fa). Inphase data

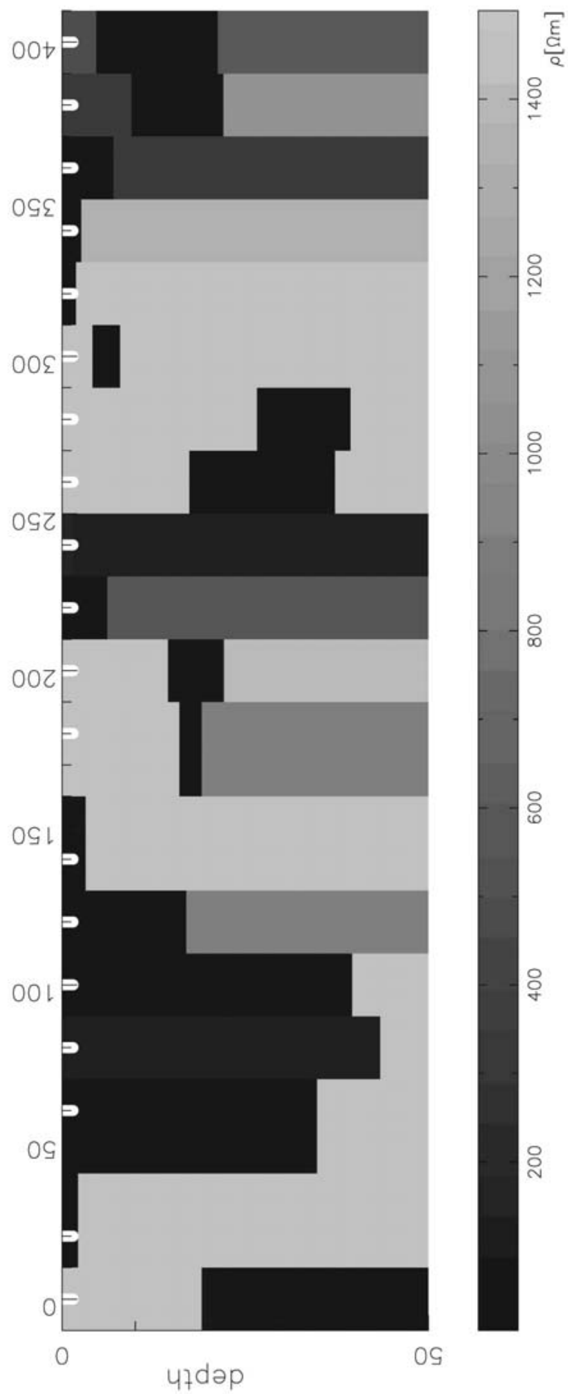

Fig. 5. Inverted resistivity section along profile B as obtained from one-dimensional inversion

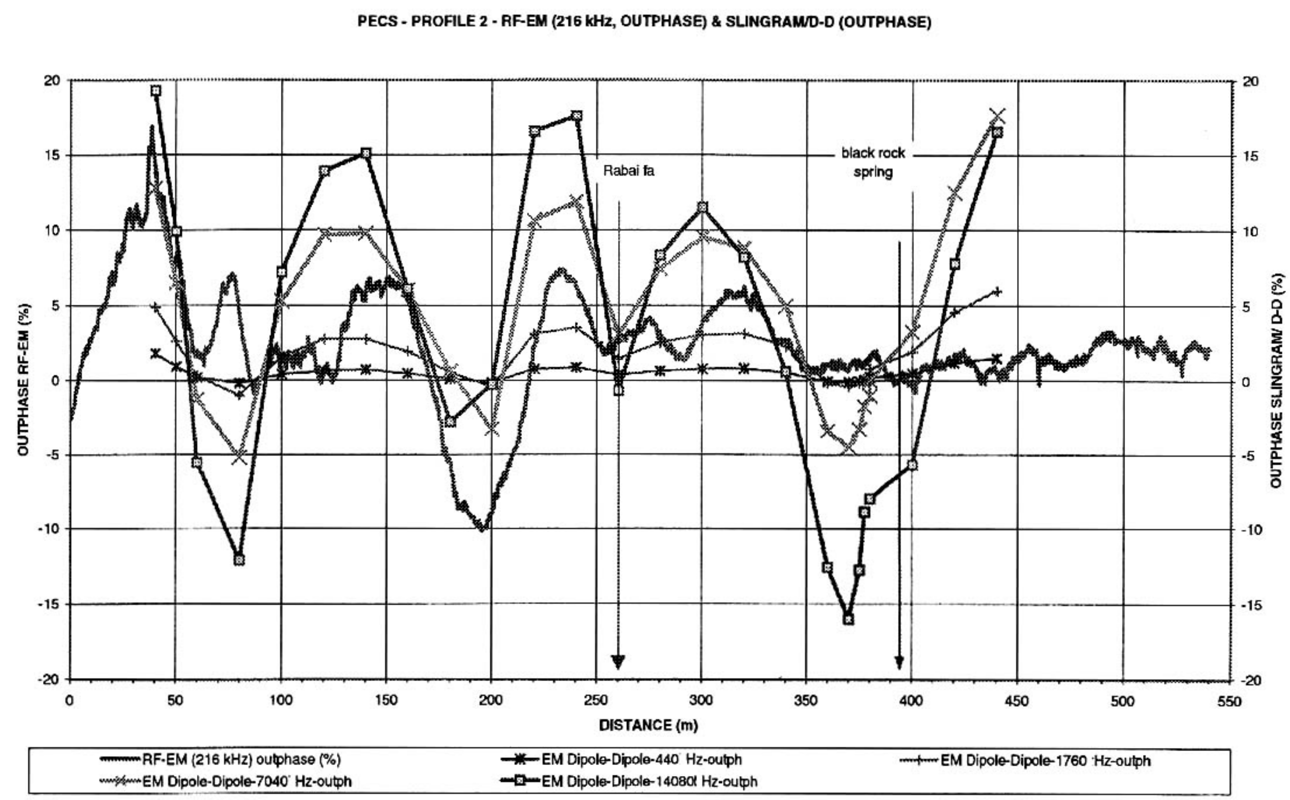

Fig. 6. RF-EM out-of-phase data along profile B at 216 kHz, as compared to Slingarm out-of-phase data at different frequencies

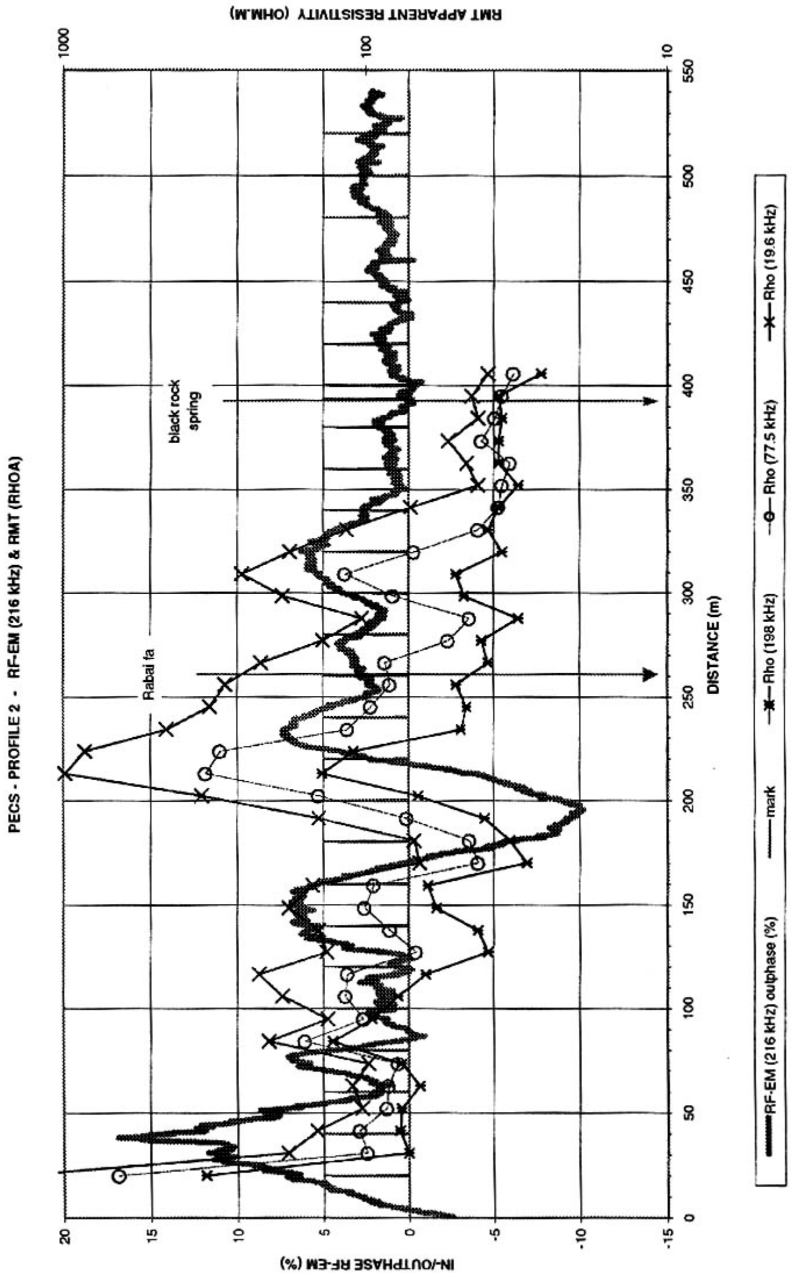

Fig. 7. RF-EM out-of-phase data along profile B at 216 kHz, as compared to radio-magnetotelluric (RMT) apparent resistivities

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