Tensor decomposition examples by using the corrected phase deviation formulas

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Introduction

In a recent paper (Práčser and Szarka, 1999) it has been shown that a correction of the phase deviation method for tensor decomposition provides more understandable and interpretable results than the original formulas published in the paper by Bahr (1991).

In this paper a systematic numerical experiment is given by using the corrected phase deviation formulas, in presence of circular (C) and elongated (E) near-surface bodies. In both cases the disturbing body is considered as an electric dipole.

The tensor decomposition with the corrected phase deviation formulas perfectly recovers the phase values of the original 2D impedance tensor.

Bahr’s (1991) Phase Deviation Method

The measured magnetotelluric tensor $Z$ in the coordinate system of the regional 2D structure:

$$Z = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

where

$$a = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}$$

is a real, frequency-independent matrix, representing the distortion due to small, localized, near-surface anomaly; $Z_{xy}$ and $Z_{zy}$ are the principal impedances for the 2D regional structure; $i$ is the deviation angle.

In the field — instead the aforementioned $Z$ — a rotated tensor is measured. It means that the first step of the decomposition is to find the rotation angle $\alpha$ together with the phase values and the second step is to determine the absolute values of the elements of $Z$.

Step 1

The correct rotation angle (Práčser and Szarka, 1999) is as follows:

$$\tan(2\alpha_{ij}) = \frac{1}{2} \left( a_{ij} + a_{ij} + c_{ij} g \right)$$

$$= \frac{1}{\lambda_{ij}(1 - a_{ij} + c_{ij} g)}$$

where

$$\lambda_{ij} = (a_{ij} - 1)(a_{ij} - 1)$$

(1) $a_{ij} = (a_{ij} - 1)(a_{ij} - 1)$

(2) $a_{ij} = (a_{ij} - 1)(a_{ij} - 1)$

(3) $a_{ij} = (a_{ij} - 1)(a_{ij} - 1)$

(4) $a_{ij} = (a_{ij} - 1)(a_{ij} - 1)$

(5) $a_{ij} = (a_{ij} - 1)(a_{ij} - 1)$

(6) $a_{ij} = (a_{ij} - 1)(a_{ij} - 1)$

and the two commutators between the complex numbers $C_1$ and $C_2$ are as follows:

$$\left[ C_1, C_2 \right] = \text{Im}(C_1C_2) = \text{Re}C_1 \text{Im}C_2 - \text{Re}C_2 \text{Im}C_1$$

$$\left[ C_2, C_1 \right] = \text{Re}(C_2C_1) = \text{Re}C_1 \text{Im}C_2 + \text{Re}C_2 \text{Im}C_1$$

(7) $\left[ C_1, C_2 \right] = \text{Im}(C_1C_2) = \text{Re}C_1 \text{Im}C_2 - \text{Re}C_2 \text{Im}C_1$

(8) $\left[ C_2, C_1 \right] = \text{Re}(C_2C_1) = \text{Re}C_1 \text{Im}C_2 + \text{Re}C_2 \text{Im}C_1$

In this first step the values of $\Phi_{xy}$, $\Phi_{zy}$ and $\delta$ are obtained as well.

Step 2

The second step of the decomposition is the solution of Equation (1), where the phase values are already known. It is worth noting that the four scalar equations of the four absolute values give solutions for four real unknowns.

In order to get undistorted $Z_{xy}$ and $Z_{zy}$, it is inevitable to know the static shift values (two values from $a_{11}, a_{22}, a_{32}$ and $a_{33}$).

For the particularisation of the field situation, the rotational invariant parameters $\mu$ (the regional one-dimensional indicator) and $\eta$ (the regional skew) are used:

$$\mu = \frac{\left( |a_{11}| + |a_{22}| \right)^{1/2}}{|a_{33}|}$$

$$\eta = \frac{\left( |a_{11} - a_{22}| \right)^{1/2}}{|a_{33}|}$$

$\eta > 0$ means that the structure is three-dimensional. According to field experiences this decomposition method is worth applying, when $\mu$ and $\eta$ differ from 0, and $\mu > \eta$. It means that both two- and three-dimensional effects are present, and the two-dimensional effect dominates.

Test of the method on synthetic examples

The effectiveness of this tensor decomposition method is illustrated on two-dimensional numerical modeling computation, by using the finite element method (Uchida and Ogawa 1993). The two-dimensional model is shown in Fig. 1.

The effect of a small and circular body is equivalent to the effect of an electric dipole, the orientation of which is the same as that of the inducing electric field (Fig. 2). It means that in E polarization it is parallel to the $x$ axis, and in H polarization it is parallel to the $y$ axis. The computation of this dipole field was done in the same way as by Groom and Bailey (1991).

In case of elongated disturbing body (Fig. 3), the direction of the substituting electric dipole does not change very much with the direction of the inducing field, as is shown in Figure 4, taken from Szarka et al. (1994).

![Figure 1. The two-dimensional model](image1)

![Figure 2. Near-surface circular local heterogeneity, distorting the MT results over the two-dimensional model](image2)

![Figure 3. Near-surface elongated local heterogeneity, distorting the MT results over the two-dimensional model](image3)

![Figure 4. Relation between the external field direction and the observed field within differently elongated thin sheet models (Szarka et al., 1994)](image4)
Figure 5. The original two-dimensional (a), the distorted- (b), the rotated- (c), and the corrected polar diagrams (d) for example C1

Figure 6. The original two-dimensional (a), the distorted- (b), the rotated- (c), and the corrected polar diagrams (d) for the example C2

Figure 7. The original two-dimensional (a), the distorted- (b), the rotated- (c), and the corrected polar diagrams (d) for example C3

Figure 8. The original two-dimensional (a), the distorted- (b), the rotated- (c), and the corrected polar diagrams (d) for example E1

Figure 9. The original two-dimensional (a), the distorted- (b), the rotated- (c), and the corrected polar diagrams (d) for example E2

Figure 10. The original two-dimensional (a), the distorted- (b), the rotated- (c), and the corrected polar diagrams (d) for example E3

References


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