Geoelectric mapping of near-surface karstic fractures by using null arrays

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ABSTRACT

The term “null array” is introduced for those electrode configurations where the measured potential difference is zero above a homogeneous half-space when using a measuring dipole M0N0. Different types of null arrays (three-electrode, Schlumberger, and dipole axial/equatorial null arrays) and their corresponding traditional arrays are studied. It was shown in a field study carried out in a karstified limestone area covered by thin sediments that it is possible to obtain geologically meaningful results with null-array techniques. The main features of the null-array data are as follows. (1) Null-array data appear to be more spatially variable than the classical data. The spatial variability provides information about the presence of karstic fractures in the subsurface; (2) The null-array anomalies caused by nearly vertical karstic fractures in the limestone basement do not decay with depth as quickly as the classical array anomalies. (3) The strike direction of the fractures is much less ambiguous than that found by using classical arrays. Nevertheless, the depth variation of the basement is more reliably observed in geoelectric anomalies obtained using traditional arrays. Therefore a joint use of classical arrays and their corresponding null methods is recommended, because the combined methods provide more information about the subsurface structure.

INTRODUCTION

If current electrodes A, B and potential electrodes M, N lie on the surface of a homogeneous half-space, depending on the mutual positions of A, B, M, and N, the measured potential difference may be zero in many places. Such configurations can be defined as “null arrays.” In this paper, we discuss null arrays with small-size MN. In such cases, null arrays are those configurations where the measuring dipole is perpendicular to the direction of the local earth current flowline.

Although there have been some occasional studies using such configurations in the 1960s–1970s (e.g., Jakosky, 1960; Gupta and Bhattacharya, 1963; Habberjam and Watkins, 1967; Brizzolari and Bernabini, 1979; Militzer et al., 1979), in addition to related studies that were carried out later (Mathias and Habberjam, 1984; Tsokas et al., 1997), the null arrays we propose in this paper have not been previously investigated. These arrays are closely related to the classical AMNB approaches differing only in that the small-size measuring dipole MN is rotated by 90° (denoted in this new position as M0N0), and that any small variations in potential difference ΔU_{M0N0} are due to lateral conductivity inhomogeneities. The field application of this idea emerged for first time following analog modeling experiments in Hungary (Szalai, 1993; Szarka, 1994).

Szalai (1993) first considered geoelectric null arrays when studying the geoelectric effect of leakages in waste-disposal-site lining using a multielectrode configuration. Szarka (1994) proposed incorporating electromagnetic configurations into the geophysical interpretation that provide zero response over a homogeneous half-space, but concrete results have been published only for different non-dc situations (Szarka, 1991; Szarka and Nagy, 1992; Szarka and Menvielle, 1999).

We are not aware of any field application of our proposed null-array technique. We believe this may be because null-array data require a completely different interpretation compared to data from traditional arrays. For example, a good
understanding of parameter-sensitivity maps is required. These maps are described in detail by Barker (1979) and by Spitzer (1998).

In this paper, we initially provide a simple classification of possible geometrical null arrays, which bring together traditional and nontraditional array pairs. The results of a case study in the Swiss Jura mountains are then presented.

**NULL ARRAY AND TRADITIONAL ARRAY PAIRS**

The potential difference is zero wherever the small measuring dipole \( M^0N^0 \) is at right angles to the local direction of the earth current flow line. Among the infinite number of arrangements, we discuss here only null arrays having corresponding configurations with the family of traditional arrays. These pairs are presented in Figure 1.

In case (I), \( MN (M^0N^0) \) is close to one of the current electrodes. In this case, the current lines are nearly radial around the current electrode. According to the similarity with the classical three-electrode (pole-dipole) configuration \( AMN \), the \( AM^0N^0 \) configuration is called the “three-electrode null array.”

In case (II), \( MN (M^0N^0) \) is located between the two current electrodes. Here, the current lines are nearly parallel. A special situation in this group is the Schlumberger (or \( AM^0N^0B \)) null array, when the center of \( M^0N^0 \) is on the line between \( A \) and \( B \) at an equal distance from both current electrodes.

In case (III), \( MN (M^0N^0) \) is far from the current electrodes. In this case, the current source may be considered as a dipole. Two null arrays can be easily constructed from well-known traditional configurations: (IIIA) the dipole axial null array and (IIIB) the dipole equatorial null array. Due to the reciprocity theorem these last two null arrays are equivalent, whereas the traditional dipole-equatorial and dipole-axial configurations give different responses.

Figure 2 shows a parameter sensitivity map for the dipole-axial null array. It was calculated using the analytical approach of Szalai and Szarka (2000). The values of isolines in Figure 2 are the percentage of effect caused by a small cube in the given place (at a depth of one tenth of the dipole-dipole distance) by this null array with respect to the homogeneous half-space value in the field of the corresponding traditional array. The antisymmetry about the \( y = 0 \) axis indicates that any anomalous body which is symmetrical over this axis has zero effect on the null-array response. In case of a symmetrical body, the effects of pairs due to related small portions (those at distances \( +y/R \) and \( -y/R \)) are mutually cancelled out. This holds true for all one-dimensional bodies (layers), two-dimensional bodies whose dip direction coincides with the \( y = 0 \) axis, and three-dimensional bodies having a symmetry axis coincident with \( y = 0 \).

**THE TEST AREA AND THE TECHNICAL PARAMETERS**

The test area is situated in the folded part of the Swiss Jura in the vicinity of a quarry near the village of Les Breuleux.
in the northwestern part of Switzerland. In this region, Kimmeridgian limestone with a thickness of several hundred meters is covered by a thin (0.2–1.5 m) layer of Quarternary clay and silt. The limestone is karstified and tectonically influenced by a combination of faults and overthrusts (Zwahlen and Doerfliger, 1995). Karstification and tectonics have generated a system of fractures and fissures. In the test area, the fractures are nearly vertical, and the fracture planes are perpendicular to the wall of the quarry. Neither the ground surface nor the limestone’s top surface is horizontal.

Measurements were carried out along three profiles (1, 2, and 3) parallel to the wall (at distances of 2, 8, and 16 m from the wall, respectively). The results obtained along profile 2 are presented in this paper. This particular distance was close enough to the wall to allow a direct comparison between the visible fractures and the obtained anomalies. On the other hand, it was far enough from the wall not to be greatly influenced by the void space. Furthermore, the profile has a gentle topography. (Even in profile 1, the data were found to be more disturbed by the rugged topography than by the proximity of the wall.) The parameters of the four arrays used are shown in Table 1.

In order to avoid any mispositioning of the electrodes, a special tool (a wooden lattice fixed on a tube) was prepared (shown in Figure 3), allowing the geometric error to be reduced to a value of less than 1 cm. Its weight was less than 4 kg, and the cables run through the tube, thereby facilitating rapid measurements in the field.

**NULL-ARRAY PROFILES AND THEIR INTERPRETATION**

A major objective of this study is to determine whether it is possible to obtain information about fractures by using non-traditional null arrays. In order to avoid the possibility of misinterpretation, test measurements were carried out to study repeatability. Since the test produced coherent results (that is, the main features of null-array profiles consistently remained the same), we proceeded to the detailed study.

**Main features of null-array anomalies**

Figure 4 presents typical apparent resistivity profiles (measured along profile 2 with the dipole-axial null array and with the traditional dipole-axial array). The depth to the limestone’s top surface (as determined by boreholes along profile 2) is also indicated. Both apparent resistivity profiles seem to be a superposition of a slowly varying trend and many small, localized undulations. For the traditional arrays, it is evident that the trend is due to the nonparallelism between the nearly flat ground surface and the top of the high-resistivity limestone basement. (See the basement depth values shown in Figure 4.) It is our hypothesis that the local variations are due to fractures in the limestone. Such an interpretation is possible also for the null arrays, as it is demonstrated by a simple logical approach summarized in Figure 5. The observed null-array anomaly is composed of steps A, B, C and D as follows.

**Step A** In case of a simple one-dimensional earth (e.g., a horizontal interface between a homogeneous high-resistivity basement and the sediment; see Figure 5a), the

<table>
<thead>
<tr>
<th>Array</th>
<th>Characteristic distance (m)</th>
<th>OAB or OAN (MN) distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>AO = 2</td>
<td>1</td>
</tr>
<tr>
<td>II</td>
<td>AB = 4</td>
<td>1</td>
</tr>
<tr>
<td>IIIA and IIIB</td>
<td>OABO = 2.5</td>
<td>1</td>
</tr>
</tbody>
</table>
null-array response on the ground surface should be zero (See also A in Figure 4a).

Step B) In case of a dipping basement (see Figure 5b), when AO is in the strike direction, a constant $\Delta U_{MNO}$ value is measured due to the perfect two dimensionality of the problem. (See also B in Figure 4a.) This $\Delta U_{MNO}$ value—due to the asymmetry about line AO—differs from zero.

Step C) If the thickness between the ground surface and the dipping high-resistivity basement increases due to a gently sloping surface (Figure 5c), the measured $\Delta U_{MNO}$ will diminish progressively, because the response is less and less influenced by the basement. In other words, the half-space seems to be more and more homogeneous for an observer at the surface. It is the case in Figure 4a (shown by curve C) in the direction of increasing distances starting at about 10 m. From this point towards zero (towards the left), another decrease is seen. Without doubt, the limestone surface is closest to the surface at 10 m. (This particular feature is not shown in Figure 5.)

Step D) The local variations (denoted as D in Figure 4a) are assumed to be a result of fractures, as shown in Figure 5d. This was later verified by numerical modeling.

The anomalies obtained by the traditional arrays are straightforward; thus, no detailed comments will be made. Nevertheless, one can identify the features B, C, D for traditional array profile as well (see B, C, D in Figure 4b; A is a nonzero constant).

**Numerical modeling results**

If vertical fractures are laterally extensive and have a horizontal upper surface, they would not have any effect on the...
null-array response. In our case, the roof of the vertical fractures coincides with the roof of the basement. An individual fracture is modeled by a thin and long (0.2 m x 10 m) low-resistivity (10 ohm-m) body. Because of the restriction of our three-dimensional numerical modeling software (Prášer, 1999; based on the approach of Dey and Morrison, 1979; for details see Appendix A), the dipping-plane upper surface was approached by staircase. The bedrock resistivity was set to 1000 ohm-m. The model is shown in Figure 6; the numerical modeling results are shown in Figure 7.

In Figure 7, the response of the body is given in millivolts/amp for both the traditional array (left side) and the corresponding null array (right side). The zero horizontal distance (directly above the center of the anomalous body) is the reference point of each array, indicated by the filled star in Figures 7a–d. Note the sharper response of the body when using the null-arrays I, IIIA, and IIIB, and the antisymmetric response relative to the traditional response using null array II (the Schlumberger null array).

Such high-conductivity fractures are indicated with null-arrays I, IIIA, and IIIB by minima in the null-array response. In the two latter cases, the theoretical responses are equivalent due to the reciprocity theorem (see Figures 7c and 7d).

INTERPRETATION OF FIELD RESULTS

Comparison of geoelectric results with the photo of the quarry wall

The measured data along profile 2 are shown in Figure 8 for the three-electrode and Schlumberger arrays and in Figure 9 for the axial and equatorial dipole-dipole arrays. The quarry wall is shown at the bottom of both figures. (Note that the depth to the limestone roof along profile 2 is shown in Figure 4.)

The slowly varying trend in the responses is due to the relative dipping of the ground surface with respect to the limestone bed: the depth to the basement increases to the right, which is shown as “basement depth” in the Figure 4. (This variation of the basement depth is only partly visible in the photo.)

By a visual comparison of the fractures on the photo with the measured profiles, we infer that the minima of the three-electrode null-arrays data give a picture of the presence of karstic fractures (e.g., at 18, 26–27, and 38 m). The Schlumberger null array in Figure 8 also seems to indicate the fractures by the minima instead of the inflection points shown by numerical results in Figure 7b. For the type of structures modeled, the distance between the inflection point and the minimum is only about 1 m, so this phenomenon, at least in this field study, is not a source of confusion.

The dipole-equatorial anomalies are slightly shifted. The dipole-axial null array again gives a poorer result than the three-electrode or the Schlumberger null array, but remains better than its corresponding traditional array, mainly on the right side of the profile.

The reason for the large peak at distance about 10 m in case of the Schlumberger null array (and for a somewhat similar peak with the dipole axial null array) is associated with the shallowest depth to the limestone roof. The enhanced position of the limestone surface is evident from shallow drillings carried out along profile 2 (shown in Figures 4a and 4b). At the same time, this enhanced position is not seen on the photo, but is hidden by the irregular wall edge.

The similarity between the dipole-equatorial and the dipole-axial null-array anomalies measured in the field (Figure 10) is apparent. This is expected because of their reciprocity relationship. Of course, the two curves are no longer identical as they were previously in the synthetic model (shown in Figures 7c and 7d) because of measuring (positional) errors.

Without doubt, the null-array data are spatially more variable than the classical data. Nevertheless, we should emphasize that they are not artifacts: they reflect the physical response from a heterogeneous ground. (They are repeatable in time, and furthermore, most minima are shown by at least two measured values.) Of course, not all of the minima could be identified as fractures, but this increased spatial variability (provided the sampling distance is small enough) may be a result of the possible presence of smaller fractures. At the same time, the increased spatial variability may obscure long-range trends in the depth-variation of the limestone roof.

As Figures 8 and 9 show, the null-array provides useful information about the presence of karstic fractures. It must be emphasized that the null-array anomalies due to the fractures do not decay with depth as quickly as the classical array anomalies do. At the same time, the classical arrays better describe the basement depth. Consequently, the joint use of null arrays and their classical array pairs is recommended.

Determination of the fracture direction by using null arrays

In order to be able to determine the fracture direction, an analog modeling experiment was carried out.

Assume an elongated horizontal and tabular low-resistivity anomalous body in the subsurface with an undetermined orientation. In Figure 11, azimuthal response profiles for the classical Schlumberger array and for the Schlumberger null array are shown for two different offsets that is the distance $d$ between the profile and the center of the body. The diagrams are results of analog modeling experiments. Rotating the array through 360° on the surface (equivalently, rotating the body in the host medium) produces, in the case of the classical array, two minima in the azimuthal response profile, whereas the

![Fig. 6. A simplified fracture model used for interpretation of array responses. The anomalous body is elongated and highly conducting; it has an infinite depth extent and a stepped upper surface. The direction of the profile is at right angles to the long axis of the body. The body is embedded in a homogeneous half-space.](image-url)
null-array has four minima. In the special case of zero offset, the direction of the body unambiguously can be determined by using the classical Schlumberger array. In the case of the Schlumberger null array, the number of minima is doubled, depending on whether AB or MN is parallel to the strike. Fortunately, this feature (according to the modeling results) will be preserved in case of nonzero offsets (that is, if we are not exactly over the center of the body). Note that in the case of nonzero offset, the direction of the body can no longer be determined by using the classical array) because the minimum zones of the azimuthal response profiles become too wide. Although some information about the direction of the body can still be obtained from the relatively narrower maximum zone, the determination of the strike direction is ambiguous. At the same time, the direction can be still reliably determined from the null-array measurements, due to the very narrow minimum zone. The remaining uncertainty of 90° is resolved with the aid of the classical Schlumberger-array measurements, which provide orientation.

The field results obtained at 17 m along profile 2 are shown in Figure 12. The elongated bodies in this case are the nearly vertical karstic fractures in the limestone. In case of the classical Schlumberger array, instead of $U_{MN}$, the transformed quantity $[(\Delta U_{MN}/I) - 16 \text{ ohms}]$ is shown. (In this way, the maximum and minimum values are seen more clearly.) As in the analog modeling experiment, the strike direction can be determined relatively unambiguously from the maximum, resulting in an estimate of $-15°$ to $165°$. For the Schlumberger null array within $360°$, again two minimum pairs are obtained: $0°$ to $180°$ and $90°$ to $270°$. The true fracture direction can be easily selected from the two possible ones, in despite of the slight ($15°$) discrepancy between the classical and the null-array version.

![Figure 7](image-url)

Fig. 7. Numerical modeling results for the classical arrays and their corresponding null arrays I, II, IIIA, and IIIB. The MN and M$^0$N$^0$ potential differences (for the traditional and the null array accordingly) are plotted as a function of the distance from the anomalous body. Note the difference of points of reference between the traditional and null arrays which are indicated by stars. (a) I, (b) a special case of II (the traditional Schlumberger array and the Schlumberger null array) (c) IIIA, (d) IIIB (its null-array version is in a reciprocity relation with that of IIIA).
strike-direction determination results. As expected, direct fracture-direction determination in the field agreed with the null-array estimation.

In Figure 13, the fracture directions as observed on the quarry wall and as determined along profile 2 by using this combined traditional/null-array resistivity technique are presented. A good correlation was found between the observed and the measured strike directions. See especially the characteristic direction of \(20^\circ\) on the left side and roughly \(0^\circ\) elsewhere.

**SUMMARY**

Different pairs of geoelectric null arrays and their classical counterparts were tested. Systematic field measurements were carried out in a karstic area. It has been shown that it is possible to get geologically meaningful results using null arrays. The null-array anomalies, because of their physically meaningful spatial variability, provide useful information in finding vertical fractures in limestone basement. It is especially worth emphasizing that the null-array anomalies do not decay
Fig. 9. Comparison of the null and classical array responses by an actual example: (a) dipole-axial null array and dipole-axial array, (b) dipole-equatorial null array and classical dipole-equatorial array, (c) the wall of the quarry.

with depth as quickly as the classical array anomalies do. At the same time, by using null arrays the strike direction of the fractures is given by sharper and narrower minimum zones with azimuthal response profiles than are obtained using classical arrays. Therefore, joint use of classical arrays and their corresponding null arrays provides complementary information about subsurface structure.

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Fig. 10. The dipole-equatorial and the dipole-axial null-array responses along profile 2. The patterns are similar due to the reciprocity theorem.

Fig. 11. Master diagrams for the determination of the strike direction by using a Schlumberger array (left) and a Schlumberger null array (right) obtained from analog modeling experiments carried out for two different offsets $d = 0$ and $d = 5$ cm.

Fig. 12. Azimuthal diagrams for the determination of the strike direction in the field by using a classical Schlumberger array (left) and a Schlumberger null array (right). In case of the classical Schlumberger array a special value ($\Delta U_{MN/I} = 16$ ohms) is shown. In this way of representation, the minimum resistance value approaches zero.


**APPENDIX A**

**OUR NUMERICAL MODELING SOFTWARE**

The software developed by Prácsér (1999) is based on the approach by Dey and Morrison (1979).

The differential equation describing the potential $U(x, y, z)$ due to a point source in three-dimensional case is

$$\text{div}\{\sigma(x, y, z)\text{grad} U(x, y, z)\} = - I \delta(x_0, y_0, z_0). \quad (A-1)$$

This equation is solved for the nodes of a three-dimensional grid. Among a node and its the six neighboring nodes is the linear relationship (Dey and Morrison, 1979).

$$\int\int\int_{V_{i,j,k}} \text{div}(\sigma \text{grad} U) \, dx \, dy \, dz = \int\int_{S_{i,j,k}} \sigma \text{grad} U \, ds. \quad (A-2)$$

The surface planes of the body $V_{i,j,k}$ are the median planes of distances connecting the node $(i, j, k)$ with its neighbors, and $S_{i,j,k}$ is the total surface of this body. As boundary conditions, the well-known boundary condition for the surface and the boundary condition for the domain boundaries is applied as

$$\alpha U + \beta \text{grad} U = \gamma, \quad (A-3)$$

where $\mathbf{n}$ is a unit vector perpendicular to the domain boundary, and the values of $\alpha$, $\beta$, and $\gamma$ can be determined from value of $U$ in the case of a homogeneous half-space. Equations (A-2) and (A-3) yield a system of linear equations, and the elements of its solution vector are the values of the potential function in the nodes.

The Fortran code is solved in iterations, using the preconditioned conjugated gradient method.